Generalized Modal Shock Spectra with Indeterminate Interface

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The generalized modal shock spectra method has proved to be an effective tool in reducing the analysis effort and degree of dependence between the spacecraft and launch vehicle design processes. However, the method has limitations on the degree of structural static determinancy of the spacecraft to launch vehicle interface. These practical limitations are removed in the present work by using the "interface modes." The governing differential equations are first derived and are shown to be valid for either integrating the modal models of two or more substructures that have been previously obtained separately, or for removing a substructure from a previously available system modal model. These equations facilitate investigating the effects of interchanging payloads on the systems response. Emphasis is placed here on the integration problem in the context of the generalized modal shock spectra approach. A minimum of information about the modal response of a previously obtained transient analysis of the unloaded launch vehicle is exploited to define an idealized modal impulse function and to derive approximate explicit expressions for an estimate of the bounds of the spacecraft response. A numerical example is solved to compare the present results with the transient solution.

Nomenclature

A_a	= acceleration at a
c	= spacecraft damping matrices, Eqs. (2), (13), and (15)
C_{ba}	= force coefficients
$d_{a/b}$	= relative motion between a and b
$D_{a/b}$	= bound on $d_{a/b}$
D_1, D_2	= response, Eqs. (19) and (20)
F_a	= components of member loads
F_I, F_J	= forces at locations I and J
F_ℓ	= launch vehicle modal force, Eq. (15)
I	= unit matrix
k	= spacecraft stiffness matrices, Eqs. (2), (13), and (15)
m	= spacecraft mass matrices, Eqs. (2), (13), and (15)
M_{RR}	= rigid mass of launch vehicle, Eqs. (8) and (9)
q_i, (q_n or q_s)	= spacecraft coordinates, interface and elastic
Q_N, Q_R	= launch vehicle coordinates, elastic and rigid
Q_i, Q_s	= bounds on q_i and q_s
$Q_{s\ell}$	= bound on participation of a mode pair s,ℓ
t	= time
$V_{0\ell}$	= magnitude of initial velocity
W	= weighting function
$X(t)$	= deformation time history, Eqs. (16) and (18)
α, β, θ	= variables, Eqs. (17) and (23)
μ	= mass ratio, Eq. (15)
ξ	= modal damping, Eqs. (3), (8), (15), (17), and (19)
ρ	= damped frequencies, Eq. (19)
τ	= $\omega_s t$
ϕ, Φ	= spacecraft and launch vehicle modes, Eqs. (2) and (13)
$\omega, \Omega, \Delta\Omega$	= frequencies, Eqs. (3), (8), (15), and (17)

Subscripts

A	= arbitrary physical point
a,b	= arbitrary degrees of freedom
i	= interface modes
$I, \bar{I}$	= interface degrees of freedom and their total number
J	= degrees of freedom at which external loads are applied
ℓ, L	= free-free launch vehicle modes (rigid or elastic); their total number is L
n	= substructure normal modes
N	= launch vehicle elastic normal modes
res	= residual
$s, S, \bar{S}$	= elastic modes of spacecraft restrained at all I degrees of freedom, total number of s is S or $\bar{S}$, depending on whether they are truncated or not

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Introduction

PREVIOUS surveys[1,2] have investigated the advantages and disadvantages of various proposed methods for the analysis and design of spacecraft structures. The methods fall into two categories, the first of which includes "full-scale" as well as "abbreviated" transient analysis approaches[3-5] which aim to either exactly or approximately simulate the flight conditions and predict the time histories of the response quantities of interest. In the second category are the shock spectra methods,[6,7] which seek to estimate bounds on the response. Of particular interest here is the generalized modal shock spectra method.[7]

The generalized modal shock spectra approach differs substantially from the traditional shock spectra definition. The most notable differences are the assumptions implied in the traditional shock spectra that 1) the mass and stiffness of the oscillator are infinitesimally small such that they do not alter the base input, and 2) the oscillator's interface to the base is determinate. To overcome these weaknesses, the derivation begins with the differential equations governing the motion of two or more substructures attached at a common statically indeterminate interface(s). If the object of the analysis were to detach the substructures, the same equations can be used with minor sign change. The impedance between the substructures (spacecraft and launch vehicle) is preserved as each spacecraft mode is coupled with each launch vehicle mode. Unlike Ref. 7, the spacecraft modes include the normal as well as the interface modes. The number of interface modes is equal to the number of interface degrees of freedom. The degree of interface indeterminancy is immaterial, and the determinate problem becomes a special case. The mathematical bases and computational approximations leading to expressions for bounds on the spacecraft displacements, accelerations, and member loads are developed in the following sections.

Basic Equations

The equation of motion for a typical substructure that is temporarily detached from the remainder of the structural system at their common indeterminate interface I, can be written in modal coordinates in the general form:

$$\begin{bmatrix} m_{ii} & m_{in} \\ m_{ni} & m_{nn} \end{bmatrix}\begin{Bmatrix} \ddot{q}_i \\ \ddot{q}_n \end{Bmatrix}+\begin{bmatrix} c_{ii} & c_{in} \\ c_{ni} & c_{nn} \end{bmatrix}\begin{Bmatrix} \dot{q}_i \\ \dot{q}_n \end{Bmatrix}$$
$$+\begin{bmatrix} k_{ii} & 0 \\ 0 & k_{nn} \end{bmatrix}\begin{Bmatrix} q_i \\ q_n \end{Bmatrix}=\begin{bmatrix} \phi_{iI} \\ 0 \end{bmatrix}\{F_I\}+\begin{bmatrix} \phi_{iJ} \\ \phi_{nJ} \end{bmatrix}\{F_J\} \quad (1)$$

Equation (1) stems from a redefinition of the modes used in Ref. 8 in that no distinction is made here between the rigid body and constraint modes. Two sets of modes are employed in Eq. (1); n normal vibration modes of the substructure in question for which all I interface degrees of freedom are restrained to zero motion, and i interface modes during which a unit deformation is imposed one at a time at each of the I interface degrees of freedom while keeping all other $(I-1)$ interface degrees of freedom restrained. The i interface modes thus obtained are linear combinations of the substructure rigid body modes and constraint modes used in Ref. 8. The statically determinate case is naturally a degenerate special case of the indeterminate one.

As a consequence to the choice of modal normal and interface coordinates described above, the following relationships hold for the submatrices of Eq. (1):

ϕ_{iI}	= unit matrix
$m_{ii}=\phi_{ia}m_{aa}\phi_{ai}$	= mass matrix associated with the interface modes
$m_{nn}=\phi_{na}m_{aa}\phi_{an}$	= mass matrix associated with the normal modes, normalized to unity
$m_{ni}=m_{in}^T=\phi_{na}m_{aa}\phi_{ai}$	= mass coupling between interface and normal modes
$c_{ii}=\phi_{ia}c_{aa}\phi_{ai}$	= damping matrix associated with the interface modes
$k_{ii}=\phi_{ia}k_{aa}\phi_{ai}$	= stiffness matrix of the interface modes; becoming zero for a determinate interface
m_{aa},c_{aa},k_{aa}	= physical mass, damping, and stiffness defined at arbitrary degrees of freedom, a
$\phi_{ai}=\phi_{ia}^T,\phi_{an}=\phi_{na}^T$	= respectively are the interface and normal mode shapes at degrees of freedom, a

(2)

The relationships defining the remaining matrices such as c_{nn}, k_{nn}, and c_{ni} are similar to their counterparts in Eqs. (2) above. Because the interface degrees of freedom are restrained during the normal mode solution, the stiffness coupling $k_{ni}=k_{in}^T=0$.

The basic equations (1) and (2) for a typical substructure are used next to synthesize the equations governing a composite structural system consisting of two substructures: a spacecraft and launch vehicle having a common interface I.

For the spacecraft there usually are no externally applied loads, except through the common interface. Designating quantities associated with the interface modes by subscript i, and those associated with the spacecraft normal modes (restrained at all I) by subscript s, and idealizing the structural damping by a diagonal matrix with the normal mode damping ξ_s as percent of the critical modal damping, Eq. (1) is specialized for the spacecraft as follows:

$$\begin{bmatrix} m_{ii} & m_{is} \\ m_{si} & I \end{bmatrix}\begin{Bmatrix} \ddot{q}_i \\ \ddot{q}_s \end{Bmatrix}+\begin{bmatrix} c_{ii} & 0 \\ 0 & 2\xi_s\omega_s \end{bmatrix}\begin{Bmatrix} \dot{q}_i \\ \dot{q}_s \end{Bmatrix}$$
$$+\begin{bmatrix} k_{ii} & 0 \\ 0 & \omega_s^2 \end{bmatrix}\begin{Bmatrix} q_i \\ q_s \end{Bmatrix}=\begin{Bmatrix} [I] & F_I \\ & 0 \end{Bmatrix} \quad (3)$$

The c_{ii} term in Eq. (3) is useful in modeling distributed structural damping as well as discrete damping mechanisms throughout the spacecraft structure, including the interface.

The definition of terms in Eqs. (2) applies to both Eqs. (1) and (3), along with the premise that all modes $i=1,2,...,\bar{I}$ and $s=1,2,...,\bar{S}$ are to be included. Although the total number of interface modes $\bar{I}$ is usually limited (less than a hundred), the total number of normal modes $\bar{S}$ can be very large (thousands). For the latter, it is not practical or desirable to retain all $\bar{S}$. Instead, only a truncated set $S<\bar{S}$ may be retained so that the dynamic analysis cost can be reduced substantially, with little loss in the accuracy of computed responses. This is done here by extending the concept of residual mass for statically determinate structures[9] to ones with indeterminate supports. First, it is observed that each spacecraft mode s contributes an effective mass matrix m_{ii}^s at the $\bar{I}$ interface degrees of freedom. From Eqs. (2), the $\bar{I}\times\bar{I}$ effective mass matrix is

$$m_{ii}^s=m_{is}m_{si} \qquad (i=1,2,...,\bar{I}) \quad (4)$$

If all $\bar{S}$ normal modes are retained, the sum of all the effective mass matrices of Eq. (4) will be equal to the total spacecraft $\bar{I}\times\bar{I}$ mass matrix obtained directly from the second of Eqs. (2). Thus,

$$m_{ii}=\phi_{ia}m_{aa}\phi_{ai}=\sum_{s=1}^{\bar{S}}m_{ii}^s \quad (5)$$

The proof of Eq. (5) stems from the definition of m_{ni} of Eqs. (2), and the orthogonality condition $m_{nn}=I$. If only S modes are retained, $S<\bar{S}$, a residual mass matrix $(m_{ii})_{\text{res}}$ corresponding to the truncated modes $(\bar{S}-S)$ will be missing from the selected spacecraft's vibration modes,

$$(m_{ii})_{\text{res}}=m_{ii}-\sum_{s=1}^{S}m_{ii}^s \qquad S\leq\bar{S} \quad (6)$$

However, the dynamic effect of $(m_{ii})_{\text{res}}$ can be partially taken into account by augmenting the mass of the interface modes m_{ii} by $(m_{ii})_{\text{res}}$. Thus, m_{ii} in Eq. (3) may be replaced by $m_{ii}+(m_{ii})_{\text{res}}$.

It should be noted that, since, in general, $\bar{I}>6$, the $(\bar{I}\times\bar{I})$ terms of m_{ii}^s, $(m_{ii})_{\text{res}}$, and m_{ii} will not have the usual meanings of mass, moment of mass, and moment of inertia. If the equivalent (6×6) rigid mass matrix for any of the above $(\bar{I}\times\bar{I})$ matrices is required with respect to an arbitrary point A, it can be computed from transformations of the type:

$$(m_{ii}^s)_{AA}=\phi_{Ai}m_{ii}^s\phi_{iA} \quad (7)$$

where $(m_{ii}^s)_{AA}$ is the m_{ii}^s equivalent (6×6) rigid mass matrix with respect to A and ϕ_{Ai} the rigid body transformation of the

$\bar{I}$ interface degrees of freedom with respect to the six degrees of freedom at A.

The launch vehicle analysis is assumed to have been already performed in a free-free condition with the common interface I to the spacecraft unloaded. This is the usual case in practice and is not a requirement of the present approach. For a free-free system, the interface modes i, defined earlier, become rigid body modes, since all interface degrees of freedom are unrestrained. As such, the launch vehicle is represented by free-free normal and rigid body modes, here designated by subscripts N and R, respectively. Furthermore, just like (F_J), the interface reaction forces $(-F_I)$ are considered external to the launch vehicle. With the normal modes normalized to unit mass, $M_{NN}=I$, $K_{NN}=\omega_N^2$, damping is represented by ξ_N percent of the critical modal damping, and $K_{RR}=C_{RR}=0$, so that Eq. (1) becomes for the launch vehicle alone:

$$\begin{bmatrix} M_{RR} & 0 \\ 0 & I \end{bmatrix}\begin{Bmatrix} \ddot{Q}_R \\ \ddot{Q}_N \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 2\xi_N\omega_N \end{bmatrix}\begin{Bmatrix} \dot{Q}_R \\ \dot{Q}_N \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \omega_N^2 \end{bmatrix}\begin{Bmatrix} Q_R \\ Q_N \end{Bmatrix} = \begin{bmatrix} \Phi_{RI} & \Phi_{RJ} \\ \Phi_{NI} & \Phi_{NJ} \end{bmatrix}\begin{Bmatrix} \mp F_I \\ F_J \end{Bmatrix} \tag{8}$$

The absence of the coupling term M_{RN} in Eq. (8) is due to the use of free-free modes. Also, instead of m_{ii} of Eqs. (2), the (6×6) rigid mass matrix M_{RR} for the launch vehicle is

$$M_{RR}=\Phi_{Ra}M_{aa}\Phi_{aR} \tag{9}$$

The equations governing the system of spacecraft and launch vehicle connected together at I are found by combining Eqs. (3) and (8) with the conditions of compatibility of deformation, velocity, and accelerations. For example, compatibility of motion at the interface requires that the following relationship and its time derivatives hold true:

$$[I]q_i=\Phi_{iR}Q_R+\Phi_{iN}Q_N \tag{10}$$

Conditions of force equilibrium at the interface are already satisfied by using F_I with the proper sign in both Eqs. (3) and (8). The use of $(-F_I)$ in Eq. (8) has the effect of adding the spacecraft, while $(+F_I)$ has the effect of removing it. The above leads to

$$F_I=[m_{ii}+(m_{ii})_{\text{res}}](\Phi_{iR}\ddot{Q}_R+\Phi_{iN}\ddot{Q}_N)+c_{ii}(\Phi_{iR}\dot{Q}_R+\Phi_{iN}\dot{Q}_N) + k_{ii}(\Phi_{iR}Q_R+\Phi_{iN}Q_N)+m_{is}\ddot{q}_s \tag{11}$$

Combining Eqs. (3), (8), (10), and (11) results in eliminating the interface quantities q_i and F_I as independent unknowns, while fully accounting for their effects in the equations governing the complete system.

$$\begin{bmatrix} (M_{RR}\pm m_{RR}) & \pm m_{RN} & \pm m_{Rs} \\ \pm m_{NR} & (I\pm m_{NN}) & \pm m_{Ns} \\ \pm m_{sR} & \pm m_{sN} & \pm I \end{bmatrix}\begin{Bmatrix} \ddot{Q}_R \\ \ddot{Q}_N \\ \ddot{q}_s \end{Bmatrix} + \begin{bmatrix} \pm c_{RR} & \pm c_{RN} & 0 \\ \pm c_{NR} & (2\xi_N\omega_N\pm c_{NN}) & 0 \\ 0 & 0 & \pm 2\xi_s\omega_s \end{bmatrix}\begin{Bmatrix} \dot{Q}_R \\ \dot{Q}_N \\ \dot{q}_s \end{Bmatrix} + \begin{bmatrix} \pm k_{RR} & \pm k_{RN} & 0 \\ \pm k_{NR} & (\omega_N^2\pm k_{NN}) & 0 \\ 0 & 0 & \pm\omega_s^2 \end{bmatrix}\begin{Bmatrix} Q_R \\ Q_N \\ q_s \end{Bmatrix} = \begin{Bmatrix} \Phi_{RJ}\ F_J \\ \Phi_{NJ}\ F_J \\ 0 \end{Bmatrix} \tag{12}$$

In addition to terms previously defined, the following relationships are used in Eq. (12):

$$\begin{aligned} &m_{RR}=\Phi_{Ri}[m_{ii}+(m_{ii})_{\text{res}}]\Phi_{iR} && c_{NN}=\Phi_{Ni}c_{ii}\Phi_{iN} \\ &m_{NN}=\Phi_{Ni}[m_{ii}+(m_{ii})_{\text{res}}]\Phi_{iN} && c_{RN}=c_{NR}^T=\Phi_{Ri}c_{ii}\Phi_{iN} \\ &m_{RN}=m_{NR}^T=\Phi_{Ri}[m_{ii}+(m_{ii})_{\text{res}}]\Phi_{iN} && k_{RR}=\Phi_{Ri}k_{ii}\Phi_{iR} \\ &m_{Ns}=m_{sN}^T=\Phi_{Ni}m_{is} && k_{NN}=\Phi_{Ni}k_{ii}\Phi_{iN} \\ &c_{RR}=\Phi_{Ri}c_{ii}\Phi_{iR} && k_{RN}=k_{NR}^T=\Phi_{Ri}k_{ii}\Phi_{iN} \end{aligned} \tag{13}$$

Notice that Φ_{Ri} and Φ_{Ni} are the launch vehicle rigid and normal mode values only at the interface degrees of freedom. For the determinate case, $k_{RR}=k_{NN}=k_{RN}=0$.

The coupled system of $(R+N+S)$ equations in Eq. (12) (with the plus sign) govern the motion of the integrated system of spacecraft and launch vehicle. If there were more than one spacecraft to be attached to more than one set of interfaces of the same launch vehicle, the last row and column sets designated by s subscripts in Eq. (12) will need to be replaced by as many row and column sets as there are spacecraft $S_1, S_2, \ldots$, etc. If the object of the synthesis is to *remove* (rather than attach) the spacecraft from a modal analysis of the launch vehicle, which has been initially performed with the spacecraft attached to their common interface, the negative signs in Eq. (12) will hold true. The validity of Eq. (12) for either adding or removing the modal models of various substructures is particularly useful when investigating the effect of interchanging payloads is of interest, as is the case for the Shuttle Orbiter.

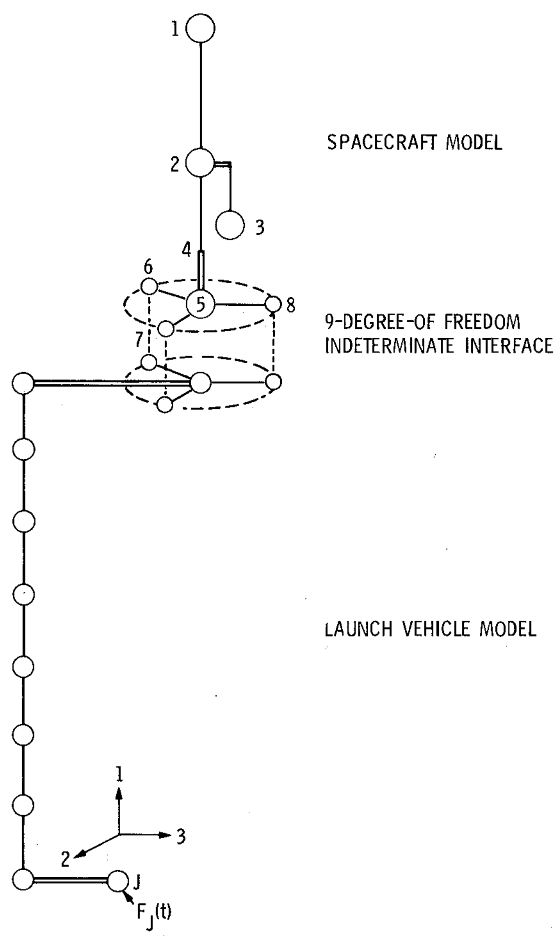

Fig. 1 Spacecraft and launch vehicle model for the numerical example (═══ rigid elements, ─── flexible elements).

A direct or modal transient solution of the coupled Eq. (12) is in principle straightforward, but numerically burdensome. Coupling between R, N, and S modes is a consequence of these being obtained from initially separate modal analyses, one for each substructure. Decoupling Eq. (12) can be done by first solving for their $(R+N+S)$ eigenvalues and eigenvectors. If the spacecraft is to be removed, one will find that, out of the $(R+N+S)$ eigenvalues and vectors, two S roots are repeated. The repeated roots identify the modes of the substructure to be removed. Of course there may be additional repeated roots due to symmetry or other physical properties of the substructure, regardless of whether one is adding or removing it.

Generalized Modal Shock Spectra

Idealized Equations

Since the interest here is in a more economical solution approach, a transient solution of Eq. (12) will not be pursued in favor of an approximate estimation of solution bounds. Following the procedure of Ref. 7, the approximate solution of Eq. (12) is achieved by idealizing the mathematical space of $(R+N+S)$ modal coordinates by an array of nested $[(R+N)\times S]$ discrete mathematical subspaces. This is physically viewed as pairing each spacecraft mode with each of the launch vehicle modes, rigid or normal, one pair at a time. As such, any coupling that exists between the launch vehicle modes is neglected, and $m_{RN}=c_{RN}=k_{RN}=0$. However, coupling between any pair of spacecraft and launch vehicle modes is not neglected. Therefore, for attaching together a typical pair of spacecraft and launch vehicle modes, Eq. (12) reduces to:

$$\begin{bmatrix} (1+\mu_{s\ell}) & \mu_{s\ell} \\ \mu_{s\ell}^T & \mu_{s\ell} \end{bmatrix} \begin{Bmatrix} \ddot{X}_\ell \\ \ddot{X}_s \end{Bmatrix} + \begin{bmatrix} (2\xi_\ell\omega_\ell + c_{\ell\ell}) & 0 \\ 0 & 2\xi_s\omega_s \end{bmatrix} \begin{Bmatrix} \dot{X}_\ell \\ \dot{X}_s \end{Bmatrix} + \begin{bmatrix} (\omega_\ell^2 + k_{\ell\ell}) & 0 \\ 0 & \omega_s^2 \end{bmatrix} \begin{Bmatrix} X_\ell \\ X_s \end{Bmatrix} = \begin{Bmatrix} \Phi_{\ell J} F_J \\ 0 \end{Bmatrix} \quad \begin{matrix} s=1,2,\ldots,S \\ \ell=1,2,\ldots,(R+N) \end{matrix} \tag{14}$$

where

ℓ = designate any launch vehicle mode; rigid mode with $\omega_\ell=0$, or normal mode with $\omega_\ell \neq 0$

$$m_{s\ell} = \Phi_{\ell i} m_{ii}^s \Phi_{i\ell}$$

$$M_{s\ell} = M_{RR} + \Phi_{Ri}(m_{ii})_{\text{res}}\Phi_{iR} \quad (\ell = \text{rigid mode})$$

$$= 1 + \Phi_{Ni}(m_{ii})_{\text{res}}\Phi_{iN} \quad (\ell = \text{normal mode})$$

$$\mu_{s\ell} = m_{s\ell}/M_{s\ell}, \quad \omega_\ell = \omega_N/\sqrt{M_{s\ell}}, \quad \xi_\ell = \xi_N/\sqrt{M_{s\ell}}$$

$$c_{\ell\ell} = c_{RR}/M_{s\ell} \text{ or } c_{NN}/M_{s\ell}$$

$$k_{\ell\ell} = k_{RR}/M_{s\ell} \text{ or } k_{NN}/M_{s\ell}$$

$$X_\ell = M_{s\ell}Q_\ell$$

$$X_s = (M_{s\ell}/\sqrt{m_{s\ell}})q_{s\ell}, \quad q_{s\ell} = q_s(s,\ell)$$

$$F_\ell(t) = \Phi_{\ell J}F_J(t) \tag{15}$$

The general form of Eq. (14) for statically indeterminate interfaces is identical to its counterpart for a determinate interface, except for the redefinition of terms and the presence of two new ones; $c_{\ell\ell}$ and $k_{\ell\ell}$.

Modal Bounds

A bound on the spacecraft's generalized modal displacement X_s in a typical (ℓ,s) subspace represented by Eq. (14) is derived from its explicit solution in which the actual modal forcing function $F_\ell(t)$ is idealized by an equivalent one having a simple variation with time. The selected idealization of $F_\ell(t)$ is an impulse delta function with magnitude $F_{0\ell}$, or, alternatively, an initial velocity having a magnitude $v_{0\ell}$. This idealization is justified on the basis that it is the bound on the response that is more meaningful for design purpose than the detailed time history of the response. The determination of $v_{0\ell}$ is the same as in Ref. 7, and therefore will not be repeated here. The explicit analytical solution of Eq. (14) is:

$$X_\ell(t) = \frac{v_{0\ell}}{\Omega_s^2 - \Omega_\ell^2}\left[\frac{\Omega_s^2(1-\Omega_\ell^2)}{\bar{\omega}_{0\ell}}\exp(-\xi_{0\ell}\omega_{0\ell}t)\sin\bar{\omega}_{0\ell}t - \frac{\Omega_\ell^2(1-\Omega_s^2)}{\bar{\omega}_{0s}}\exp(-\xi_{0s}\omega_{0s}t)\sin\bar{\omega}_{0s}t\right]$$

$$X_s(t) = \frac{\Omega_\ell^2\Omega_s^2 v_{0\ell}}{(\Omega_s^2-\Omega_\ell^2)}\left[\frac{1}{\bar{\omega}_{0\ell}}\exp(-\xi_{0\ell}\omega_{0\ell}t)\sin\bar{\omega}_{0\ell}t - \frac{1}{\bar{\omega}_{0s}}\exp(-\xi_{0s}\omega_{0s}t)\sin\bar{\omega}_{0s}t\right] \tag{16}$$

The following parameters have been used in Eqs. (16).

$$\omega_{0\ell} = \omega_s\Omega_\ell, \quad \omega_{0s} = \omega_s\Omega_s$$

$$\Omega_\ell = \Omega_0 - (\Delta\Omega/2), \quad \Omega_s = \Omega_0 + (\Delta\Omega/2)$$

$$\Omega_0 = \frac{1}{2}\sqrt{\left(\frac{1}{\omega_s}\sqrt{\omega_\ell^2 + k_{\ell\ell}} + 1\right)^2 + \mu_{s\ell}}$$

$$\Delta\Omega = \sqrt{\left(\frac{1}{\omega_s}\sqrt{\omega_\ell^2 + k_{\ell\ell}} - 1\right)^2 + \mu_{s\ell}}$$

$$\bar{\omega}_{0\ell} = \omega_{0\ell}\sqrt{1-\xi_{0\ell}^2}, \quad \bar{\omega}_{0s} = \omega_{0s}\sqrt{1-\xi_{0s}^2}$$

$$\xi_{0\ell}\omega_{0\ell} + \xi_{0s}\omega_{0s} = 2\omega_s\beta, \quad \xi_{0\ell}\omega_{0\ell} - \xi_{0s}\omega_{0s} = 2\omega_s\theta$$

$$\beta = \frac{1}{2\omega_s}[\xi_\ell\omega_\ell + 0.5c_{\ell\ell} + (1+\mu_{s\ell})\xi_s\omega_s]$$

$$\theta = \left[\frac{(\xi_\ell\omega_\ell + 0.5c_{\ell\ell})(1-\Omega_\ell^2)^2 + \xi_s\omega_s\mu_{s\ell}\Omega_\ell^4}{\omega_s(1-\omega_\ell^2)^2 + \mu_{s\ell}} - \beta\right] \tag{17}$$

Since $\xi_{0\ell}^2$ and ξ_{0s}^2 are much smaller than unity, and $\theta<\beta$, the solution X_s may be reduced to a simpler form:

$$X_s(\tau) \cong v_{0\ell}\sqrt{\omega_\ell^2 + k_{\ell\ell}}[D_1(\tau) + D_2(\tau)] \tag{18}$$

where

$$D_1(\tau) = \frac{1}{2\omega_s^2\rho_s}\left(\frac{\rho_s}{\rho_\ell} - \theta\tau\right)e^{-\beta\tau}\sin\rho_\ell\tau\cos\rho_s\tau$$

$$D_2(\tau) = \frac{-1}{2\omega_s^2\rho_\ell}\left(\frac{\rho_\ell}{\rho_s} - \theta\tau\right)e^{-\beta\tau}\cos\rho_\ell\tau\sin\rho_s\tau$$

$$\rho_\ell = \Omega_0\sqrt{1-\xi_0^2}, \qquad \rho_s = (\Delta\Omega/2)\sqrt{1-\xi_0^2}$$

$$\xi_0 \cong (\xi_{0\ell} + \xi_{0s})/2, \qquad \tau = \omega_s t \tag{19}$$

Unlike the traditional shock spectra concept where the base input is assumed independent of the presence of the oscillator, Eqs. (14-19) allow the interaction between the spacecraft and launch vehicle modal responses $X_\ell(\tau)$ and $X_s(\tau)$. In this sense, the maximum value of $X_s(\tau)$ may be viewed as a "generalized" displacement shock spectrum. A bound $Q_{s\ell}$ on the maximum of $X_s(\tau)$ can be found directly from Eqs. (16), or may be given by the discrete maxima $D_{\max}$ of the simple sinusoid derived from Eq. (18), where $\cos\rho_\ell\tau$ and $\cos\rho_s\tau$ are either zero or unity. In the latter approach, $D_{\max}$ is the largest of

$$D_{1\max}=\frac{1}{2\omega_s^2\rho_s}\left(\frac{\rho_s}{\rho_\ell}-\theta\bar{\tau}\right)e^{-\beta\bar{\tau}}\sin\rho_\ell\bar{\tau}$$

$$D_{2\max}=\frac{1}{2\omega_s^2\rho_\ell}\left(\frac{\rho_\ell}{\rho_s}-\theta\bar{\tau}\right)e^{-\beta\bar{\tau}}\sin\rho_s\bar{\tau} \tag{20}$$

where $\bar{\tau}$ is the smallest root of

$$\tan\rho_\ell\bar{\tau}=\frac{\rho_\ell}{\beta}\frac{[1-(\rho_\ell/\rho_s)\theta\bar{\tau}]}{[1+(\rho_\ell/\rho_s)(\theta/\beta)(1-\beta\bar{\tau})]}\quad \text{for } D_{1\max}$$

$$\tan\rho_s\bar{\tau}=\frac{\rho_s}{\beta}\frac{[1-(\rho_s/\rho_\ell)\theta\bar{\tau}]}{[1+(\rho_s/\rho_\ell)(\theta/\beta)(1-\beta\bar{\tau})]}\quad \text{for } D_{2\max} \tag{21}$$

Thus, an estimate of the generalized modal displacement bound $Q_{s\ell}$ is computed from the results of Eqs. (15) and (18-21) to give

$$Q_{s\ell}=v_{0\ell}\sqrt{\mu_{s\ell}}(\omega_\ell^2+k_{\ell\ell})D_{\max} \tag{22}$$

An alternate, still simpler form of Eqs. (20-22) may be obtained by taking $\theta=0$, in which case $D_{2\max}>D_{1\max}$, and the resulting approximation for $Q_{s\ell}$, is

$$Q_{s\ell}=\frac{v_{0\ell}}{\omega_s^2}\sqrt{\frac{\mu_{s\ell}(\omega_\ell^2+k_{\ell\ell})}{\Delta\Omega^2+4\beta^2}}e^{-(2\alpha\beta/\Delta\Omega)},\quad \alpha=\tan^{-1}\left(\frac{\Delta\Omega}{2\beta}\right) \tag{23}$$

The expression for $Q_{s\ell}$ given by either Eq. (22) or (23) represents contribution of only one launch vehicle mode. The bound Q_s for a spacecraft modal response is obtained by adding contributions of all launch vehicle modes in the root-sum-square (rss) sense with a weighting function $W_{s\ell}$ as in Ref. 7.

$$Q_s=\|\,Q_{s\ell}\,\|^\ell=\left[\sum_{\ell=1}^{L}(W_{s\ell}Q_{s\ell})^2\right]^{1/2} \tag{24}$$

A significant improvement over Ref. 7 in determining $W_{s\ell}$ is achieved by using Eq. (24) in conjunction with the interface acceleration $\ddot{Q}_{0i}(t)$ resulting from a transient analysis of the launch vehicle loaded with a rigid payload. Such analysis is usually available at the early stage of spacecraft design. Thus, by neglecting the spacecraft/launch vehicle impedance—here designated by subscript 0 when $\mu_{s\ell}\to 0$ in Eq. (22) or (23)—the following resulting expression is used to determine $W_{s\ell}$ numerically:

$$|q_{0s}(t)|_{\max}=\left[\sum_{\ell=1}^{L}(W_{s\ell}Q_{0s\ell})^2\right]^{1/2} \tag{25}$$

where the left-hand side is the exact maximum modal response to the base motion $\ddot{Q}_{0i}(t)$.

In addition to the bound Q_s on the spacecraft normal mode participation, bounds Q_i and $\ddot{Q}_i$ on the interface modal response q_i and $\ddot{q}_i$ of the coupled system are needed. These can be computed from Eqs. (10) and (16) and their time derivatives, so that

$$Q_i=\Phi_{i\ell}\bar{Q}_\ell,\qquad \ddot{Q}_i=\Phi_{i\ell}\ddot{\bar{Q}}_\ell$$

$$\bar{Q}_\ell=X_\ell/M_{s\ell},\qquad \ddot{\bar{Q}}_\ell=\ddot{X}_\ell/M_{s\ell}$$

$$\ell=1,2,\ldots,L=(R+N) \tag{26}$$

and X_ℓ, $\ddot{X}_\ell$, respectively, are the maximum numerical values of $X_\ell(t)$ and $\ddot{X}_\ell(t)$ of Eqs. (16) or its approximate equivalent. Instead of Eqs. (16), the bound Q_i can be approximated, as in Ref. 7, by:

$$\ddot{Q}_i=\Phi_{i\ell}v_{0\ell}\sqrt{\omega_\ell^2+k_{\ell\ell}} \tag{27}$$

One of the significant advantages of the generalized modal shock spectra is that it allows an easy introduction of the design loads sensitivity to statistical variations in the stiffness and mass properties of the structure. This was done in Ref. 7 for the statically determinate case by introducing artificial tuning between the spacecraft and launch vehicle frequencies. The same idea is also directly applicable here when the interface is indeterminate.

Spacecraft Response Spectra

The response quantities of special interest for spacecraft design are the relative displacements between two given degrees of freedom, the absolute acceleration at certain degrees of freedom, and the load components in the structural members. Expressions for these quantities in the case of a statically indeterminate interface do not differ significantly in their structure from this counterpart for a statically determinate interface.

Relative Displacements

The relative displacement between two degrees of freedom a and b during the spacecraft motion:

$$d_{a/b}(t)=\sum_{s=1}^{S}(\phi_{as}-\phi_{bs})q_s(t)+\sum_{i=1}^{I}(\phi_{ai}-\phi_{bi})q_i(t) \tag{28}$$

However, since the time variable is not retained in the shock spectra approach, Eq. (28) must be replaced by an estimate of its bound $D_{a/b}$. This latter is computed by summation over contributions of all spacecraft modes in the rss sense, designated by $\|\ldots\|^s$ for the normal modes, and $\|\ldots\|^i$ for the interface modes.

$$D_{a/b}\cong\|(\phi_{as}-\phi_{bs})Q_s\|^s+\|(\phi_{ai}-\phi_{bi})Q_i\|^i \tag{29}$$

where Q_s and Q_i are given by Eqs. (24) and (26).

Absolute Acceleration

The absolute acceleration a_a at a given degree of freedom on the spacecraft is

$$a_a(t)=\sum_{s=1}^{S}\phi_{as}\ddot{q}_s(t)+\sum_{i=1}^{I}\phi_{ai}\ddot{q}_i(t) \tag{30}$$

Replacing $\ddot{q}_s(t)$ and $\ddot{q}_i(t)$ by their bounds $\ddot{Q}_s$ and $\ddot{Q}_i$, the acceleration bound A_a may be computed from

$$A_a\cong\|\phi_{as}\ddot{Q}_s\|^s+\|\phi_{ai}\ddot{Q}_i\|^i \tag{31}$$

Or making use of the relationship in Eq. (3) when $\dot{q}_s=0$,

$$\ddot{Q}_s+m_{si}\ddot{Q}_i+\omega_s^2Q_s=0$$

$$A_a\cong\|\phi_{as}\omega_s^2Q_s\|^s+\|(\phi_{ai}-\phi_{as}m_{si})\ddot{Q}_i\|^i \tag{32}$$

In practice, the choice between Eqs. (31) and (32) may depend on their relative conservatism.

Member Loads

Defining elements of the force coefficient C_{ba} as the bth force component associated with a unit displacement in the ath degree of freedom, one may use the modal displacement

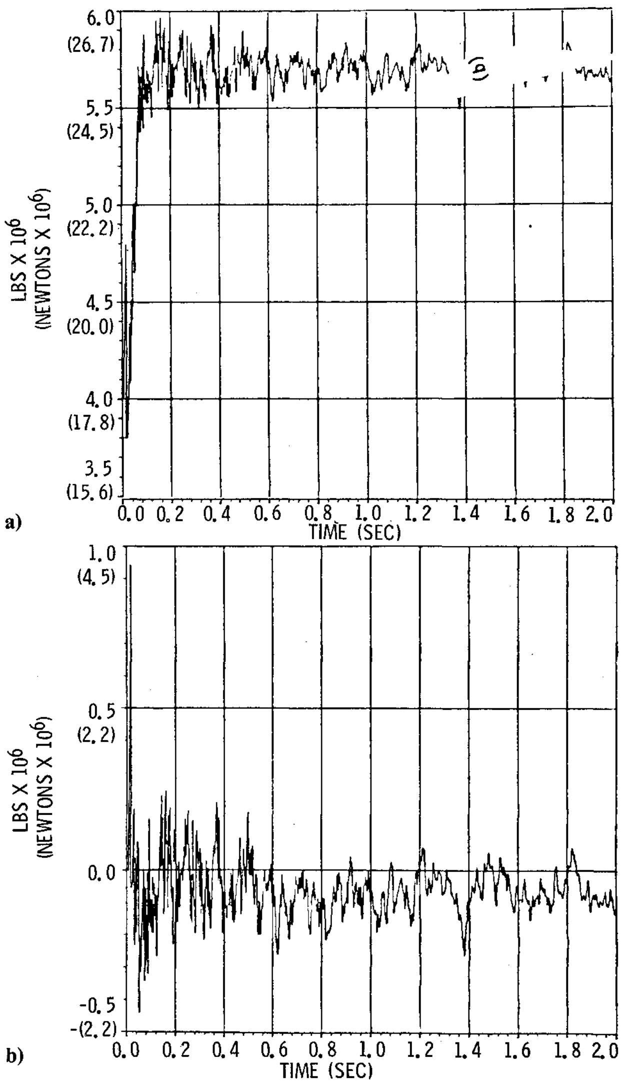

Fig. 2 Forcing function $F_J(t)$ for the numerical example. a) In longitudinal direction, 1. b) In two lateral directions, 2,3.

approach to compute the member loads from

$$F_a \cong \| C_{ba}\phi_{as}Q_s \|^s + \| C_{ba}\phi_{ai}Q_i \|^i \tag{33}$$

Alternatively, the modal acceleration method may be used with Eqs. (26), (27), and (32) to give

$$F_a \approx \| C_{ba}\phi_{as}\omega_s^{-2}\ddot{Q}_s \|^s + \| C_{ba}\phi_{as}\omega_s^{-2}m_{si}\ddot{Q}_i \|^i + \| C_{ba}\phi_{ai}Q_i \|^i \tag{34}$$

The last two parts of Eq. (34) may be further combined as coefficients of Q_i or $\ddot{Q}_i$ using Eq. (26) or (27).

Numerical Example

The purpose of this example is to demonstrate the results of the spacecraft response determination by the generalized modal shock spectra approach described above, and to compare these results with the exact transient solution. The spacecraft and launch vehicle models used are shown in Fig. 1. They represent a much simplified version of the actual Galileo spacecraft on the Shuttle-Centaur launch vehicle. The spacecraft-launch vehicle interface consists of nodes 6, 7, and 8, each having three translational degrees of freedom and giving rise to nine interface modes. In addition, M_x,M_y,M_z,I_x mass and inertia defined for each of nodes 1,2,3 give rise to 12 normal modes ranging from 23.78 to 267.3 Hz for the spacecraft restrained at the interface degrees of freedom. The total mass of the spacecraft is 0.144×10^6 kg, while that of the launch vehicle is 1.615×10^6 kg. The unloaded launch vehicle (without spacecraft) consists of six rigid body modes and 57 normal modes, all of which are free-free. The normal modes range in frequency from 1.2 to 193.0 Hz.

The free-free integrated spacecraft and launch vehicle model, having 75 rigid and normal modes, is used in a complete transient solution with the forcing functions of Fig. 2 applied at location J. The resulting maximum acceleration responses at nodes 1,2,3 on the spacecraft are shown in the "transient" column of Table 1.

Table 1 Comparison of the transient and generalized shock spectra (acceleration in g's)

Node	Degrees of freedom	Transient	Shock spectra
1	1	1.80	2.30
	2	0.33	1.03
	3	0.33	1.02
2	1	1.78	1.93
	2	0.21	0.70
	3	0.25	1.29
3	1	1.83	2.89
	2	0.39	0.74
	3	0.43	1.26

The forcing functions of Fig. 2 are also applied in a transient analysis of the free-free 63-mode model of the unloaded launch vehicle in order to obtain its modal displacement and modal acceleration time histories $Q_\ell(t)$ and $\ddot{Q}_\ell(t)$, $\ell = 1,2,...,63$. The launch vehicle dynamic characteristics, ω_ℓ, $\Phi_{i\ell}$, $\ddot{Q}_\ell(t)$, $i = 1,...,9$, $\ell = 1,...,63$, are required to determine all the quantities (including the magnitude of the idealized forcing function $v_{0\ell}$) needed to compute the spacecraft response by the shock spectra approach described in this paper. Three tunings (0, $\pm 5\%$) between the spacecraft and launch vehicle frequencies were used to obtain the individual maximum values shown in the "shock spectra" column of Table 1. By comparing the two solution approaches, Table 1 reveals that the generalized modal shock spectra approach is generally more conservative than the exact transient solution. The degree of conservatism is up to a factor of 1.5 for the large response values and up to a factor of five for the small response values.

Conclusions

The additional complexities of any degree of static indeterminancy of the spacecraft to launch vehicle interface, and the presence of discrete damping mechanisms, have been dealt with by replacing the usual spacecraft rigid body modes by the more general interface modes. Such interface modes are not needed for the free-free launch vehicle model. Thus, the method requires no additional analysis demands from the launch vehicle analyst for handling indeterminate interfaces.

The system of equations is amenable to solution by either a transient or a shock spectra approach. The generalized modal shock spectra approach pursued here still retains its cost effectiveness for the iterative design process of spacecraft structures as well as its ability to produce design loads which are less sensitive to localized design changes than the transient approach. The key to the latter property is the use of bounds rather than detailed time histories for evaluating the design quantities of interest. From the present example and from the author's experience in applying the method to the more complex actual Galileo spacecraft model, the generalized modal shock spectra approach typically incurs smaller conservatism (up to a factor of 1.5) in the large values of response quantities, but incurs larger degrees of conservatism (up to a factor of five) in the small response values. However, the degree of conservatism reduces considerably to an overall average factor of 1.5 times the transient solution, as one seeks design values that envelope the requirements of multiple design events or forcing functions.

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